Q1 Possible values of N

Thursday, November 29, 2012 9:51 AM

We will use MATLAB to find the values of N when i and j are between 0 and 7.

Here is the code:

```
[I J] = meshgrid(0:6,0:6);
N = I.^2 + I.*J + J.^2;
N = unique(reshape(N, 1, numel(N)));
N = N(N > 7);
N = N(1:15)
```

This part finds the unique values of N

Take only N > 7

Use only 15 values.

So, the next 15 values of N are

9	12	13	16	19	21	25	27	28	31	36	37	39	43	48

We know that we can't have any missing values of N between the above numbers because we have consider all i, j between 0 and 6. Any other values of N must come from (i, j) pair which has at least one of the i or j ≥ 7 which will give $N \geq 7^2 = 49$.

Q2 Simplex and Duplex

Thursday, November 29, 2012 9:54 AM

(a) Each simplex channel use 25 kHz.

So, each duplex channel use 25 × 2
= 50 kHz.

Total spectrum = 20 MHz

$$\# \text{ duplex channel} = \frac{20 \times 10^{\cancel{6}\,3}}{5\cancel{0} \times \cancel{10^3}} = 400 \text{ channels}$$

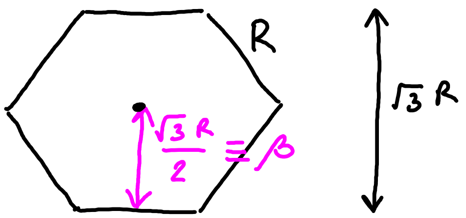
(b) Each cluster will use to whole 400 channels. These channels are divided among the cells in each cluster.

For N=4, there are 4 cells in a cluster.

Hence

$$\# \text{ channel} = \frac{400}{4} = 100 \text{ channels per cell site.}$$

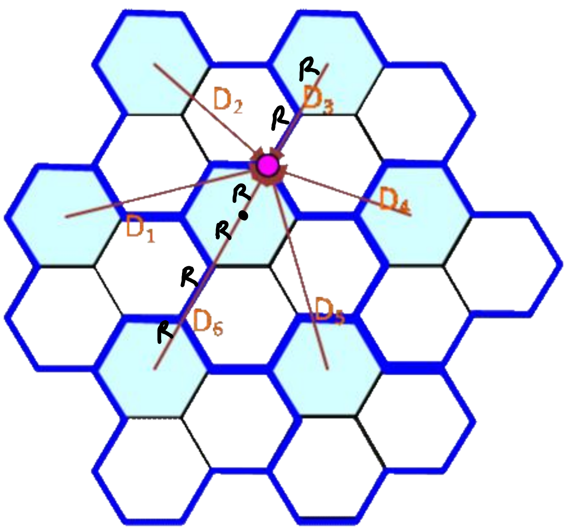
(a) To find the distance $D_1, \dots, D_6$,
let's recall some facts about hexagon.



D_3 and D_6 are easy to find. $D_3 = R + R = 2R$

$D_6 = R + R + R + R = 4R$

For the rest of the distances, the key to find them is to select suitable right triangles.



$$D_2^2 = D_4^2 = \left(\frac{R}{2}\right)^2 + (3\beta)^2 = \left(\frac{1}{4} + 9 \times \frac{3}{4}\right) R^2$$

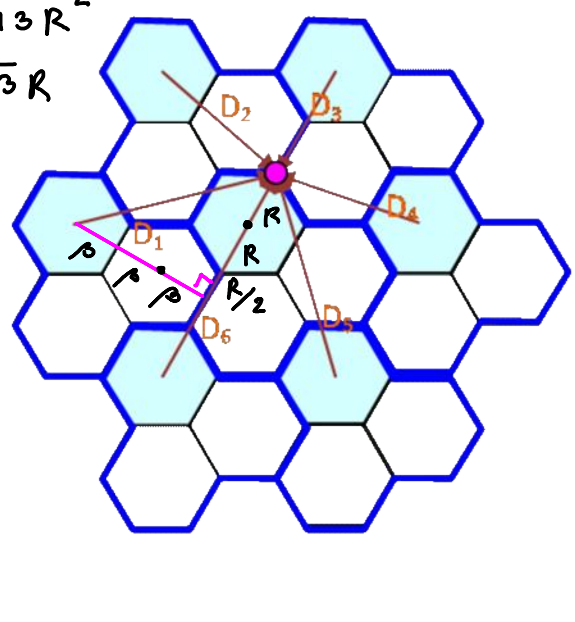
$$= 7R^2$$

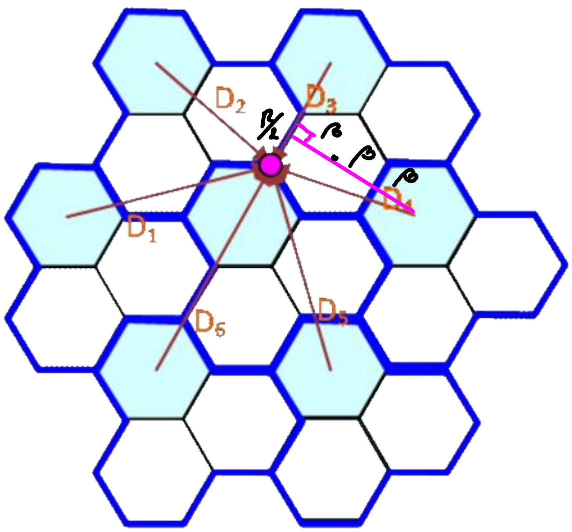
So, $D_2 = D_4 = \sqrt{7} R$

$$D_1^2 = D_5^2 = (3\beta)^2 + \left(\frac{5}{2}R\right)^2 = \left(9 \times \frac{3}{4} + \frac{25}{4}\right) R^2$$

$$= 13R^2$$

$D_1 = D_5 = \sqrt{13} R$





So,

$$D_1 = D_5 = \sqrt{13}\,R$$
$$D_2 = D_4 = \sqrt{7}\,R$$
$$D_3 = 2R$$
$$D_6 = 4R$$

(b) $$\frac{S}{I} = \frac{R^{-\gamma}}{\sum_{i=1}^{6} D_i^{-\gamma}} = \frac{1}{\sum_{i=1}^{6}\left(\frac{D_i}{R}\right)^{-\gamma}} = \frac{1}{2\times(\sqrt{13})^{-4} + 2\times(\sqrt{7})^{-4} + 2^{-4} + 4^{-4}}$$

$$= 8.399 \qquad = 10\log 8.399 \text{ dB}$$
$$= 9.242 \text{ dB}$$

$$\frac{D}{R} = \sqrt{3N} \Rightarrow D = \sqrt{3N}\times R = \sqrt{3\times 3}\,R \quad (N=3)$$
$$= 3R$$

$$\frac{S}{I} = \frac{1}{6\times 3^{-4}} = 13.5 \qquad = 10\log 13.5 \text{ dB}$$
$$= 11.303 \text{ dB}$$

(d) In part (c) we use approximated distances and hence the answer is different from part (b) which use the exact distances.

When N is large, the difference will be small.

Q4 Sectoring

Thursday, November 29, 2012 9:59 AM

$$\frac{S}{I} = \frac{1}{K}\left(\sqrt{3N}\right)^4 = \frac{1}{K}(3N)^2 = \frac{1}{K}9N^2$$

We need this number to be $\geq 15\,dB = 10^{\frac{15}{10}} = 10^{\frac{3}{2}}$

Recall that we want N to be small to get large capacity value. Hence, we need to pick minimal value of N such that the above inequality is still satisfied.

For (a), we use $K=6$.

$$\frac{S}{I} = \frac{1}{\cancel{6}_{2}} \times \cancel{9}^{3} \times N^2 \geq 15\,dB$$

$$N^2 \geq \frac{2}{3} \times 10^{\frac{3}{2}}$$

Possible values of N are 3 4 7 9 ....

$$N \geq \sqrt{\frac{2}{3} \times 10^{3/2}} = 4.591$$

From Q1, the min value of N such that it is still ≥ 4.6 is $N = 7$.

For (b), we use $K = 2$.

$$\frac{S}{I} = \frac{1}{2} \times 9N^2 \geq 10^{3/2}$$

$$N \geq \sqrt{\frac{2}{9} \times 10^{3/2}} = 2.651$$

From Q1, the min value of N such that it is still ≥ 2.6 is $N = 3$.

For (c), we use $K = 1$

$$\frac{S}{I} = 9N^2 \geq 10^{3/2}$$

$$N \geq \sqrt{\frac{1}{9} \times 10^{3/2}} = 1.874$$

From Q2, the min value of N such that it is still ≥ 1.874 is $N = 3$

$\frac{A_{system}}{A_{cell}} \frac{S}{N}$

So, by using 120° sectoring, the "capacity" of the system increases from the case of omnidirectional antenna.

However, if we've already use 120° sectoring, using 60° sectoring does not help in term of "capacity"!!

The last example in Sec. 2.4 takes this example further and instead of simply using the "capacity" defined earlier, it consider the number of users that the system can support without making the call blocking probability exceed 5%.

Q5 Erlang B

Thursday, November 29, 2012 10:03 AM

Let $$\text{ErlangB}(m, A) = \frac{A^m/m!}{\sum_{i=0}^{m} A^i/i!}$$

This gives the probability of blocking (P_b).

Ofcourse, we want P_b to be small.
In this question, we want $P_b \le \frac{0.5}{100} = 0.005$.

For fixed m, ErlangB(m, A) is an increasing function of A. Hence, if we don't want P_b to be greater than some value, we will need to limit the value of A to be less than some max quantity as well.

(a) $m = 5 \Rightarrow P_b = \text{ErlangB}(5, A) \le 0.005$

$$\Big\downarrow \text{MATLAB}$$

$A \lesssim 1.13$ Erlangs

Each user generates 0.1 Erlangs.

So n users will generate $n \times 0.1$ Erlangs.

Hence, we need $n \times 0.1 \le 1.13$

$n \lesssim 11.3$

So, the system can support 11 users

(b) $m = 15 \Rightarrow \text{Erlang B}(15, A) \leq 0.005$

$$A \leq 7.38$$

$$\Rightarrow n \leq 73.8$$

So, the system can support 73 users

(c) $m = 25 \Rightarrow \text{Erlang B}(25, A) \leq 0.005$

$$A \leq 14.997$$

$$\Rightarrow n \leq 149.97$$

So, the system can support 149 users

Q6 Erlang B

Thursday, November 29, 2012 10:04 AM

$\lambda = 3$ calls per hour

$\frac{1}{\mu} = 5$ minutes $= \frac{5}{60}$ hour $= \frac{1}{12}$ hour.

(a) $A_u = \frac{\lambda}{\mu} = 3 \times \frac{1}{12} = \frac{1}{4}$ Erlang per user

(b) Erlang B $(1, A) \leq 0.0101$

$$\Rightarrow \underset{n \times A_u}{\underset{\parallel}{A}} \leq 0.0101$$

$$n \leq \frac{0.0101}{1/4} = 0.0404$$

So, the system can support 0 user

Note that this calculation comes from our assumption of M/M/m/m queue which assumes "infinite" number of users with extremely small Erlang per user.

Of course, we never have "infinite" number of users in real system.

When the number of users is large, the Erlang B formula provides reasonable answer.

Here, the number of users is very small and the Erlang B formula gives strange answer.

Intuitively, the system which has one channel should be able to support at least 1 user with 0% blocking.

The Erlang B formula should become more accurate when there are a lot of users.

(c) $\text{ErlangB}(5, A) \leq 0.01$

$$\Rightarrow \underset{\overset{\shortparallel}{n \times A_u}}{A} \leq 1.36$$

$$n \leq 4 \times 1.36 = 5.44$$

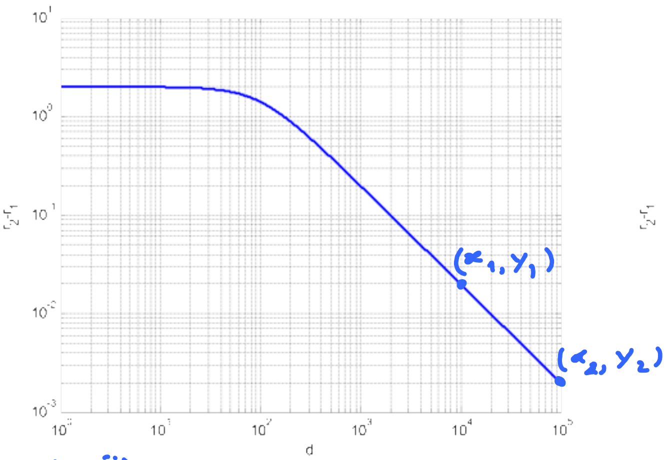
so, the system can support 5 users

(d) $\text{ErlangB}(5, \underbrace{2 \times 5}_{\text{new } n} \times \underbrace{\tfrac{1}{4}}_{A_u}) = 0.0697 = 6.97\%$

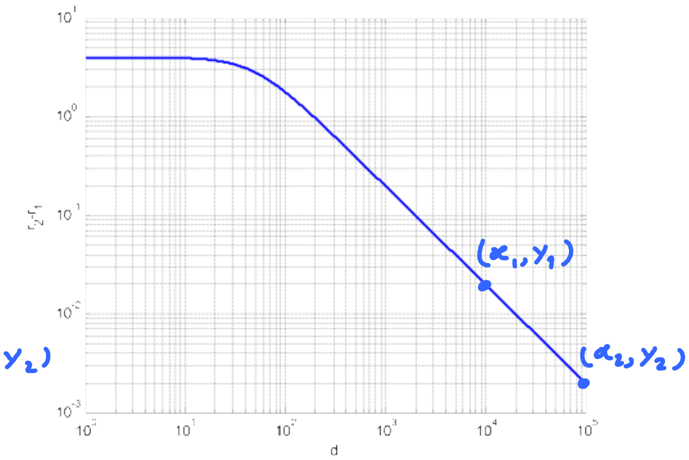
Q7 Reflection from a ground plane

Thursday, November 15, 2012 8:45 AM

(a) $h_t = 50\,m$, $h_r = 2\,m$



(a.i) $h_t = 100\,m$, $h_r = 1\,m$



(a.ii)

For large d, the two plots are virtually the same.

To find the slope, we define $y = \log_{10}(r_2 - r_1)$

$x = \log_{10} d$

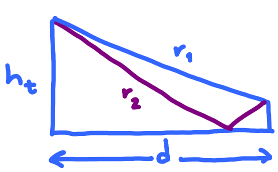
$$\text{slope} \approx \frac{y_2 - y_1}{x_2 - x_1} \approx \frac{\log_{10}(2\times10^{-3}) - \log_{10}(2\times10^{-2})}{\log_{10}(10^5) - \log_{10}(10^4)} = -1$$

Therefore, $\log_{10}(r_2 - r_1) \approx B - \log_{10} d$ for some constant B

$r_2 - r_1 \approx \frac{10^B}{d} = \frac{b}{d}$ for some constand b.

$$\Rightarrow r_2 - r_1 \propto \frac{1}{d}.$$

(b) Assume $d \gg h_t, h_r$



$r_1^2 = (h_t - h_r)^2 + d^2$

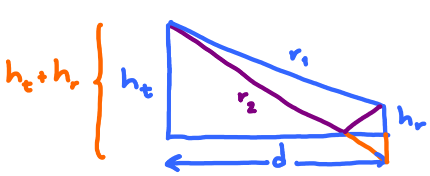
$r_1 = \sqrt{(h_t-h_r)^2 + d^2} = d\sqrt{1 + \underbrace{\left(\frac{h_t-h_r}{d}\right)^2}_{\text{small}}}$

small by the assumption

as hinted

$\approx d\left(1 + \frac{1}{2}\left(\frac{h_t-h_r}{d}\right)^2\right)$

Similarly,



$r_2^2 = (h_t + h_r)^2 + d^2$

$r_2 = \sqrt{(h_t+h_r)^2 + d^2} = d\sqrt{1 + \left(\frac{h_t+h_r}{d}\right)^2}$

$\approx d\left(1 + \frac{1}{2}\left(\frac{h_t+h_r}{d}\right)^2\right)$

as hinted

$r_2 - r_1 \approx \frac{d}{2}\left(\left(\frac{h_t+h_r}{d}\right)^2 - \left(\frac{h_t-h_r}{d}\right)^2\right) = \frac{d}{2}\,\frac{4h_t h_r}{d^2} = \frac{2h_t h_r}{d} = \frac{b}{d}$

as hinted

$$r_2 - r_1 \approx \frac{d}{2}\left(\left(\frac{h_t+h_r}{d}\right)^2 - \left(\frac{h_t-h_r}{d}\right)^2\right) = \frac{d}{2}\,\frac{4h_t h_r}{d^2} = \frac{2h_t h_r}{d} = \frac{b}{d}$$

where $\underset{\uparrow}{b} = 2h_t h_r$.

proportionality constant

Remark: The approximation "$\sqrt{1+x} \approx 1 + \frac{x}{2}$ when x is small" can be derived from the Taylor series expansion of $\sqrt{1+x}$ around 0:

Let $g(x) = \sqrt{1+x}$. Then $g(x) \approx \underbrace{g(0) + g'(0)x}_{\text{we use only this part.}} + \frac{g''(0)}{2!}x^2 + \cdots$

$g(0) = \sqrt{1+0} = 1$. $g'(x) = \frac{1}{2}(1+x)^{-1/2} \Rightarrow g'(0) = \frac{1}{2}$.

(c) (c.i)

As hinted, we will first show that for $g(t) = a_1\cos(2\pi f_c t + \phi_1) + a_2\cos(2\pi f_c t + \phi_2)$,

$$P_g = \frac{1}{2}\left|a_1 e^{j\phi_1} + a_2 e^{j\phi_2}\right|^2.$$

First, we will try to combine the two terms in $g(t)$. To do this, we write

$$g(t) = \mathrm{Re}\left\{a_1 e^{j2\pi f_c t} e^{j\phi_1} + a_2 e^{j2\pi f_c t} e^{j\phi_2}\right\}$$

$$= \mathrm{Re}\left\{\underbrace{(a_1 e^{j\phi_1} + a_2 e^{j\phi_2})}_{\substack{\text{This is simply a complex number.} \\ \text{We can write it as } a e^{j\phi}.}} e^{j2\pi f_c t}\right\}$$

$$= \mathrm{Re}\left\{a e^{j2\pi f_c t} e^{j\phi}\right\} = a\cos(2\pi f_c t + \phi).$$

In class, we know that signal of this sinusoidal form has power $P_g = \frac{1}{2}a^2$.

Observe that the number a is simply the magnitude of $a_1 e^{j\phi_1} + a_2 e^{j\phi_2}$.

Therefore, $P_g = \frac{1}{2}\left|a_1 e^{j\phi_1} + a_2 e^{j\phi_2}\right|^2$

For $y(t)$, we have $a_1 = \frac{\alpha}{r_1}\sqrt{2P_t}$, $a_2 = -\frac{\alpha}{r_2}\sqrt{2P_t}$,

$\phi_1 = -2\pi f_c \frac{r_1}{c}$, $\phi_2 = -2\pi f_c \frac{r_2}{c}$.

So, $P_y = \frac{1}{\cancel{2}} \times \cancel{2} P_t \times \left|\frac{\alpha}{r_1} e^{-j2\pi f_c \frac{r_1}{c}} - \frac{\alpha}{r_2} e^{-j2\pi f_c \frac{r_2}{c}}\right|^2$

From $x(t) = \sqrt{2P_t}\cos(2\pi f_c t)$, we have $P_x = P_t$.

Therefore, $\frac{P_y}{P_x} = \left|\frac{\alpha}{r_1} e^{-j2\pi f_c \frac{r_1}{c}} - \frac{\alpha}{r_2} e^{-j2\pi f_c r_2/c}\right|^2$

(c.ii)

① We factor out $e^{-j2\pi f_c \frac{r_1}{c}}$ to get

(c.ii)

① We factor out $e^{-j2\pi f_c \frac{r_1}{c}}$ to get

$$\frac{P_y}{P_x} = \left| e^{-j2\pi f_c \frac{r_1}{c}} \left(\frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi f_c \frac{r_2 - r_1}{c}} \right) \right|^2$$

Then use the fact that $|e^{jx}| = 1$ to get

$$\frac{P_y}{P_x} = \left| \frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi f_c \frac{r_2 - r_1}{c}} \right|^2.$$

② From part (b), we have $r_2 - r_1 \approx 2\frac{h_t h_r}{d}$

Also, $c = f_c \lambda \Rightarrow \frac{f_c}{c} = \frac{1}{\lambda}$.

③ From part (b), we have

$$r_1 - d \approx \frac{1}{2}\frac{(h_t - h_r)^2}{d} \quad \text{and} \quad r_2 - d \approx \frac{1}{2}\frac{(h_t + h_r)^2}{d}.$$

Therefore, when d is large, $r_1, r_2 \approx d$.

This allows factoring $\frac{\alpha}{d}$ out.

④ When d is large, $\frac{4\pi h_t h_r}{d}$ will be small.

For small z, we have $e^z \approx 1 + z$.

⑤ and $1 - e^z \approx 1 - (1+z) \approx -z$